14. L. Kh. Ingel', Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana, 17, No. 2, 138-145 (1981).
15. A. D. Amirov, Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana, 4, No. 2, 152-159 (1968).
16. N. A. Kudryashov and V. M. Prostokishin, Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana, 21, No. 6, 582-588 (1985).
17. G. M. Makhviladze and S. P. Sherbak, Inzh.-Fiz. Zh., 38, No. 3, 528-537 (1980).
18. A. F. Solovnik and Yu. A. Gostintsev, in: Turbulent Jet Flows [in Russian], Tallin (1985), pp. 232-238.

CALCULATION OF THE FLOW OF A POLYDISPERSED SYSTEM OF PARTICLES

A. K. Azhibekov

UDC 532.529

We discuss a model which can be used to compute the velocities and temperatures of solid and liquid particles in the presence of collisions and coagulation.

Calculation of the flow of a polydispersed system of solid and liquid particles has been considered by numerous researchers in recent years ([1-4] and others). This is because of the great abundance of multiphase systems in nature (aerosol processes in the atmosphere) and in technology (flow of a gas with suspended particles in a nozzle, carburetion processes in combustion chambers, etc.).

We consider the steady one-dimensional flow of a polydispersed system of solid and liquid particles in a gas. As an example, we consider the motion of a three-phase system consisting of a gas, solid dust particles, and water droplets, where the latter two phases are suspended in the gas. The system flows in a channel of variable cross section (a venturi serving as a dust trap). The fundamental problem is to determine the parameters of the three-phase mixture and to calculate the degree of precipitation of solid particles into the liquid droplets, the pressure drop, and the temperature decrease of the carrier medium. Obviously in order to be able to solve this problem, we must know the size distribution functions of the solid particles $dN_1 = f(\delta_1)d\delta_1$, m^{-3} and the liquid droplets $dN_2 = f(\delta_2)d(\delta_2)$, m^{-3} under a variety of conditions. The most significant factor for these distributions is the collision and coagulation of particles of different fractions. There are three types of collisions for the problem considered here: a) solid particle—solid particle collisions; b) liquid droplet—liquid droplet collisions; c) solid particle—liquid droplet collisions.

From the estimates of [4] we assume that collisions between solid particles do not lead to their coagulation; these collisions are then termed ineffective. On the other hand, the other two types of collisions are effective, and each collision leads to complete coagulation of the particles. We note that collisions of the last two types are the basic process of dust capture and therefore the degree of purification of the exhaust gas depends on the frequency and effectiveness of these collisions. Strictly speaking (as shown in [1]) a not uncommon case is when the collision leads to fragmentation of the particles, as well as partial coagulation. Processes of this type are not considered at all in the present paper.

A collision leads to an excess (or deficit) of momentum and energy of the newly formed (as a result of coagulation) particle. Therefore the velocity and temperature of the newly formed particle can differ significantly from the velocity and temperature of particles of the same size but not subjected to perturbing factors. In addition, it is very important in the solution of problems of this kind to take into account the fact that solid particles, which earlier had precipitated into liquid droplets, can return to the flow after the droplets have completely evaporated.

Alma-Atinsk Civil-Engineering Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 53, No. 6, pp. 930-936, December, 1987. Original article submitted August 25, 1986.

0022-0841/87/5306-1403$12.50

In [4-6] it was assumed that the velocity and temperature of particles formed as a result of coagulation were identical to the velocity and temperature of particles having the same size from the start of motion. But this assumption is inconsistent with the conservation laws. In addition, the return of solid particles back into the gas flow from completely evaporated droplets was not taken into account.

We explain the method and approach used in the present paper for the example of the evolution equation for the number of liquid particles $f(\delta_2)$ of a certain size δ_2 in the presence of collisions and coagulation. According to [5], we can write the distribution function in the form

$$f(\delta_2,\ x)=\frac{u_2(0)\,s(0)}{u_2(x)\,s(x)}\left[f(\delta_2,\ 0)+\int\limits_0^x\frac{u_2(x)\,s(x)}{u_2(0)\,s(0)}\,[I_1-I_2]\,\frac{dx}{u_2(x)}\right], \tag{1}$$

where I_1 and I_2 are the integrals giving the production of particles of size δ_2 from particles of sizes δ_2' and δ_2'', $\delta_2''=\sqrt[3]{\delta_2^3-(\delta_2')^3}$, and the loss of particles of size δ_2 because of collision and coagulation with particles of all other sizes δ_2^*. These integrals are given by

$$I_1=\int\limits_0^{\delta_2/\sqrt[3]{2}}k(\delta_2',\ \delta_2'')\,f(\delta_2'')\,d\delta_2', \tag{2}$$

$$I_2=f(\delta_2)\int\limits_0^{\infty}k(\delta_2,\ \delta_2^*)\,f(\delta_2^*)\,d\delta_2^*. \tag{3}$$

According to the classification adopted in [1], this approach is known as the discrete approach (abrupt changes of the state of the particle δ_2 due to collisions) and the method is the Euler method.

The velocity u_2 of the liquid particle appears in (1). In order to determine this quantity it is necessary to take into account coagulation between the droplets and also collisions of the droplets with solid particles. Strictly speaking, each collision between a solid and liquid particle leads to the formation of a new kind of particle, whose properties are different from those of the colliding particles. It would then be necessary to introduce additional distribution functions in velocity, temperature, number density, and so on, and this would significantly complicate the practical utility of the method. Therefore we do not take into account the change in the properties of a droplet when a number of solid particles are absorbed by the droplet. This assumption becomes more applicable the smaller the number and size of the solid particles in comparison with the water droplets [4].

In calculating the flow of a polydispersed system of solid and liquid particles it is not sufficient to consider only the evolution of the composition, as was done in [2, 3], for example. Rather the changes in the velocities and temperatures of the particles and the carrier medium must also be determined. In this case an important question is how the excess (or deficit) of momentum and energy of the produced particles are redistributed in the system. Two hypotheses were discused in [1, 7]. The first is that the momentum and energy excess is uniformly distributed among particles of the same fraction; the second is that the excess is rapidly transferred to the carrier medium. According to the estimates of [1, 7], preference should be given to the first hypothesis. We will use this assumption in calculating the velocities and temperatures of particles for the three types of collisions mentioned above.

In its most general formulation, the elementary theory of mechanical collisions was developed in the mid-1960's in the works of G. L. Babukhi [8]. According to this theory the one-dimensional equation of motion of a polydispersed ensemble of solid particles with collisions, but without coagulation, has the form

$$\frac{du(\delta_1,\ x)}{dx}=B_1+0{,}75\,(1-k_n)\,\frac{w}{u_1}\int\limits_0^{\infty}E_{11}\,\frac{(\delta_1+\delta_1')^2}{\delta_1^3+(\delta_1')^3}\,\frac{u_1'-u_1}{u_1'}\,|u_1'-u_1|\,(\delta_1')^3 f(\delta_1')\,d\delta_1'. \tag{4}$$

Here B_1 is the right hand side of the equation of motion of a single particle for vertical flow and is given by

$$B_1 = \frac{3}{4}\psi_1 \frac{\rho}{\rho_{sol}\,\delta_1}\frac{(w-u_1)|w-u_1|}{u_1} + \frac{g}{u_1}, \tag{5}$$

where $\psi_1 = \frac{24}{\mathrm{Re}} + \frac{4}{\sqrt[3]{\mathrm{Re}}}$ is the drag coefficient of a solid particle [4]; $k_n \in (-1; 0)$ is the coefficient of restitution of the normal component; and E_{11} is the coefficient of capture.

In view of the fact that heat transport is weak in collisions between particles that do not lead to coagulation, we can neglect it. The temperature of a particle can then be found from the heat transfer equation for a single particle [9].

Under certain conditions particles captured earlier by droplets can appear in the flow at a certain distance x from the start of motion. (Here and below we consider the case when $t > t(\delta_2)$ and $t \geq t(\delta_1)$.) We now formulate this problem. Suppose the liquid particles in the initial flow are distributed in a certain way $f(\delta_2, x) = f(\delta_2, 0)$; see Fig. 1.

At the point x_1 all droplets with sizes between zero and δ_2' are evaporated; at x_2 all droplets up to size δ_2'' are evaporated, at x_3 all droplets up to δ_2''' are evaporated, and so on. We can then construct the dependence of δ_{2ev} on the coordinate x (Fig. 2). In a segment dx droplets are evaporated which on entry had sizes between δ_{2ev} and $\delta_{2ev} + (d\delta_{2ev}/dx)dx$. During the motion, particles of size δ_1 precipitate into droplets with sizes from zero to δ_{2ev}, and the number of precipitated particles is $f(\delta_1)\int_0^{\delta_{2ev}} k(\delta_1, \delta_2) f(\delta_2)\, d\delta_2$.

It is very important to know the total number of particles of size δ_1 liberated from all evaporating drops from the start of motion. This can be written as

$$\Delta f(\delta_1) = \int_0^x f(\delta_1) \int_0^{\delta_{2ev}} k(\delta_1, \delta_2) f(\delta_2)\, d\delta_2 dx. \tag{6}$$

The solid particles returning to the flow from evaporating droplets can be included in the equation of motion (4) by introducing a correction to the velocity of a particle of size δ_1 in the form

$$B_{12} = \frac{1}{u(\delta_1)} \int_0^{\delta_{2ev}} k(\delta_1, \delta_2) f(\delta_2)\, [u(\delta_2) - u(\delta_1)] d\delta_2. \tag{7}$$

The heat transfer equation of the particle can be written in this case as

$$\frac{dt(\delta_1, x)}{dx} = \frac{\alpha(\delta_1) s(\delta_1)[t - t(\delta_1)]}{\pi/6 c_{sol}\rho_{sol}\delta_1^3 u(\delta_1)} +$$

$$+\frac{1}{u(\delta_1)} \int_0^{\delta_{2ev}} k(\delta_1, \delta_2) f(\delta_2) \left[t(\delta_2) - t(\delta_1) + \frac{1}{2c_{sol}}(u(\delta_2) - u(\delta_1))^2\right] d\delta_2. \tag{8}$$

The first term inside the square brackets in (7) is the velocity of a solid particle of size δ_1, which is equal to the velocity of a droplet of size δ_2 ($u(\delta_2)$) up to the instant of complete evaporation of the droplet. The second term is the velocity of a solid particle $u(\delta_1)$ of the same fraction δ_1 which has not precipitated into the droplet. In the heat transfer equation for the particle (Eq. (8)), the first term on the right hand side takes into account heat exchange of a single particle with the gas, and the second term is a correction to the temperature due to the "liberation" of solid particles from evaporating droplets. Similarly, the first term inside the square brackets of the second member of (8) is the temperature of a solid particle inside the droplet up to the instant of evaporation. We assume that the temperature of the particle in this case is equal to that of the droplet ($t(\delta_2)$). The second term is the temperature of a "free" solid particle of the same fraction

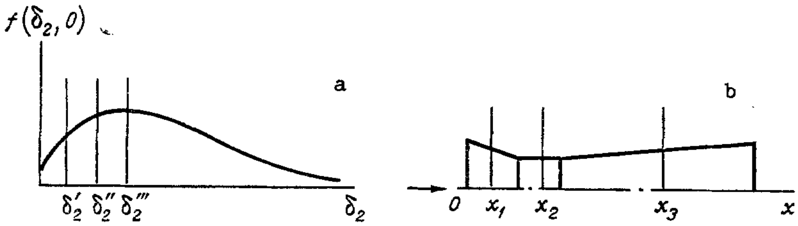

Fig. 1. Initial droplet size distribution (a) and flow pattern (b).

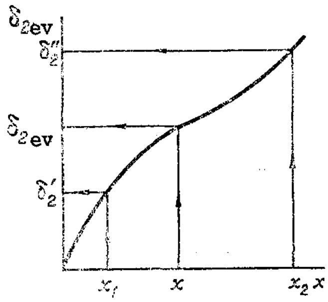

Fig. 2. Dependence of the size of an evaporating droplet on the coordinate x.

$\delta_1(t(\delta_1))$. The velocity correction (7) and the temperature correction of the particle (second term in (8)) are obtained assuming that the excess momentum $m(\delta_1)[u(\delta_2) - u(\delta_1)]$ and energy $m(\delta_1)\,[c_{\text{sol}}(t(\delta_2) - t(\delta_1)) + \frac{1}{2}(u(\delta_2) - u(\delta_1))^2]$, due to evaporation and earier captured solid particles returning to the flow, are uniformly distributed among particles with sizes between δ_1 and $\delta_1 + d\delta_1$.

In those cases when evaporation does not lead to complete disappearance of the droplet, the temperature of the particle must be determined from the heat transfer equation for a single particle interacting with the gas (the first term in (8)).

If the droplets collide as they move, and if each collision leads to coagulation, the equation of motion of a droplet of size δ_2 must be written in the form

$$\frac{du(\delta_2, x)}{dx} = B_2 + \frac{1}{u(\delta_2) f(\delta_2)} \int\limits_0^{\delta_2/\sqrt[3]{2}} k(\delta_2', \delta_2'') f(\delta_2'') f(\delta_2') \varphi(\delta_2, \delta_2') \, [U_{22} - u(\delta_2)] \, d\delta_2'. \tag{9}$$

Here B_2 is the right hand side of the equation of motion of a single droplet, and is given by

$$B_2 = \frac{3}{4} \psi_2 \frac{\rho}{\rho_{\text{liq}} \delta_2} \frac{(w - u_2)|w - u_2|}{u_2} + \frac{g}{u_2}, \tag{10}$$

where $\psi_2 = \psi_1 \left[1 + 0{,}03 \frac{\rho (w - u_2)|w - u_2| \delta_2}{\sigma}\right]^2$ is the drag coefficient of the droplet; ψ_1 is the drag coefficient of a rigid spherical droplet, and the expression in the square brackets is the correction due to flattening of the droplet; $\varphi(\delta_2, \delta_2') = [1 - (\delta_2'/\delta_2)^3]^{-\frac{2}{3}}$ is a function taking into account the nonlinearity of the relation $\delta_2^3 = (\delta_2')^3 + (\delta_2'')^3$ [5].

The equation for heat and mass transfer of a droplet, with the coagulation correction included, has the form

$$\frac{dt(\delta_2, x)}{dx} = A_2 + \frac{1}{u(\delta_2) f(\delta_2)} \int\limits_0^{\delta_2/\sqrt[3]{2}} k(\delta_2', \delta_2'') f(\delta_2') f(\delta_2'') \varphi(\delta_2, \delta_2') \left[T_{22} - t(\delta_2) + \frac{1}{2c_{\text{liq}}}(U_{22} - u(\delta_2))^2\right] d\delta_2'. \tag{11}$$

Here A_2 is the right hand side of the equation of heat and mass transfer between a single droplet of size δ_2 and the gas, and is given by

$$A_2 = \frac{\alpha(\delta_2) s(\delta_2)[t - t(\delta_2)] - r[\beta(\delta_2) s(\delta_2)[c(\delta_2) - c]]}{\frac{\pi}{6} c_{1iq} \rho_{1iq} \delta_2^3 u(\delta_2)}. \tag{12}$$

Along with coagulation of droplets, the motion of a polydispersed system of solid particles and droplets is accompanied by particle-droplet collisions and coagulation.

The equation of motion of a droplet, including collisions with solid particles, can be represented in the form

$$\frac{du(\delta_2, x)}{dx} = B_{22} + \frac{1}{u(\delta_2)} \int_0^\infty k(\delta_1, \delta_2) f(\delta_1)[U_{21} - u(\delta_2)]\, d\delta_1. \tag{13}$$

Here B_{22} is the right hand side of the equation of motion of a droplet of size δ_2 (given by (9)), including the coagulation correction due to collisions of droplets of sizes δ_2' and δ_2''.

The change of temperature of the droplet in this case can be found from the expression

$$\frac{dt(\delta_2, x)}{dx} = A_{22} + \frac{1}{u(\delta_2)} \int_0^\infty k(\delta_1, \delta_2) f(\delta_1) \left[T_{21} - t(\delta_2) + \frac{1}{2c_{1iq}}[U_{21} - u(\delta)]^2\right] d\delta_1. \tag{14}$$

Here A_{22} is the right hand side of the equation of heat and mass transfer with coagulation between droplets taken into account (given by (11)).

The first term inside the square brackets of (9) and (13) is the velocity of a droplet of size δ_2, which was either created by the coagulation of droplets of sizes δ_2' and δ_2'' (the term U_{22}) or it collided and combined with a particle of size δ_1(the term U_{21}). The second term is the velocity of a droplet of the same size δ_2 which did not undergo collisions, and is given by (10). Similarly, the first term inside the square brackets of the heat and mass transfer equations (11) and (14) is the temperature of a droplet of size δ_2 which either was created by coagulation of two smaller droplets δ_2' and δ_2'' (the term T_{22}) or it collided and combined with a particle of size δ_1(the term T_{21}). The second term is the temprature of a droplet δ_2 which did not undergo collisions (and is found from (12)).

The velocities U_{22}, U_{21} and the tempratures T_{22} and T_{21} are given by the expressions

$$U_{22} = \frac{(\delta_2')^3 u(\delta_2') + (\delta_2'')^3 u(\delta_2'')}{(\delta_2')^3 + (\delta_2'')^3}; \qquad U_{21} = \frac{\rho_{so}\delta_1^3 u(\delta_1) + \rho_{1iq}\delta_2^3 u(\delta_2)}{\rho_{so}\delta_1^3 + \rho_{1iq}\delta_2^3}; \tag{15}$$

$$T_{22} = \frac{(\delta_2')^3 t(\delta_2') + (\delta_2'')^3 t(\delta_2'')}{(\delta_2')^3 + (\delta_2'')^3}; \qquad T_{21} = \frac{c_{sol}\rho_{so}\delta_1^3 t(\delta_1) + c_{1iq}\rho_{1iq}\delta_2^3 t(\delta_2)}{c_{so}\rho_{so}\delta_1^3 + c_{1iq}\rho_{1iq}\delta_2^3}.$$

The coagulation corrections in the equations of motion (9) and (13), and in the heat and mass transfer equations (11) and (14) were obtained assuming that the collision-induced excess momentum $m(\delta_2)[U_{22} - u(\delta_2)]$ in (9) and $m(\delta_2)[U_{21} - u(\delta_2)]$ in (13) and energy $m(\delta_2)\left[c_{1iq}(T_{22} - t(\delta_2)) + \frac{1}{2}(U_{22} - u(\delta_2))^2\right]$ in (11) and $m(\delta_2)\left[c_{1iq}(T_{21} - t(\delta_2)) + \frac{1}{2}(U_{21} - u(\delta_2))^2\right]$ in (14) are uniformly distributed among droplets of sizes between δ_2 and $\delta_2 + d\delta_2$.

If the number of droplets, and therefore the probability of collision between droplets, is small, B_{22} and A_{22} in (13) and (14) will reduce to B_2 and A_2. The velocity and temperature of the drops can then be found from (10) and (12).

Equations (4), (7), (8), (9), (11), (13), and (14), together with the equations of coagulation [4, 5], heat and mass transfer [4], and the equations of motion, energy, and continuity of the polydispersed system of solid and liquid particles, and also the gas [6], form a closed system of equations. With specified boundary conditions, these equations describe the dependence of the flow parameters on the coordinate x.

NOTATION

δ_1, δ_2, size of solid particles and droplets; g, acceleration of gravity; w, gas flow velocity; ν, kinematic viscosity of the gas; ρ_{sol}, ρ_{liq}, intrinsic densities of the solid particles and liquid droplets; c_{sol} and c_{liq}, the intrinsic heat capacities of the solid particles and liquid droplets; σ, the surface tension; $u(\delta_1)$, $u(\delta_2)$ (or u_1 and u_2), velocities of solid particles and droplets; $Re=|w - u(\delta)|\delta/\nu$, Reynolds number for the relative motion of a solid particle (subscript 1) or of a liquid droplet (subscript 2) of size δ; δ_1', u_1', size and velocity of particles colliding with a particular particle of size δ_1; k, coagulation coefficient of the particles (from [1, 4], for example); t, $t(\delta_1)$, $t(\delta_2)$, temperatures of the gas, solid particles, and droplets; $\alpha(\delta_1)$, $\alpha(\delta_2)$, coefficients of heat transfer from particles and droplets, respectively, to the gas; $\beta(\delta_2)$, coefficient of mass transfer from droplets to the gas; $s(\delta_1)$, $s(\delta_2)$, the suraface of a particle and droplet, respectively; r, latent heat of vaporization; c and $c(\delta_2)$, concentration of water vapor in the gas and on the surface of a droplet.

LITERATURE CITED

1. L. E. Sternin (ed.), Monodispersed and Polydispersed Flow of a Gas with Particles [in Russian], Moscow (1980).
2. C. T. Crowe and P. G. Willoughby, AIAA J., 4, No. 9, 1677-1678 (1966).
3. F.E. Marble, Astronaut. Acta, 13, No. 2, 159-166 (1967).
4. I. B. Palatnik, B. E. Lavrov, and G. N. Kogai, Foundations of the Operation of Dust Collecting with a Venturi Coagulator [in Russian], Alma-Ata (1977).
5. I. B. Palatnik and A. K. Azhibekov, Inzh.-Fiz. Zh., 35, No. 4, 698-704 (1978).
6. A. K. Azhibekov, Civil Engineering Technology and Design [in Russian], Alma-Ata (1983), pp. 41-46.
7. A. N. Kraiko and A. A. Shraiber, Zh. Prikl. Mekh. Tekh. Fiz., No. 2, 67-74 (1974).
8. A. A. Shraiber, V. N. Milyutin, and V. P. Yatsenko, Hydromechanics of Two-Component Flows with Polydispersed Solid Material [in Russian], Kiev (1980).
9. S. L. Soo and F. E. Marble, Phys. Fluids, 8, No. 9, 1751-1753 (1965).